General Issues in Uncertainty Analysis for Complex Models

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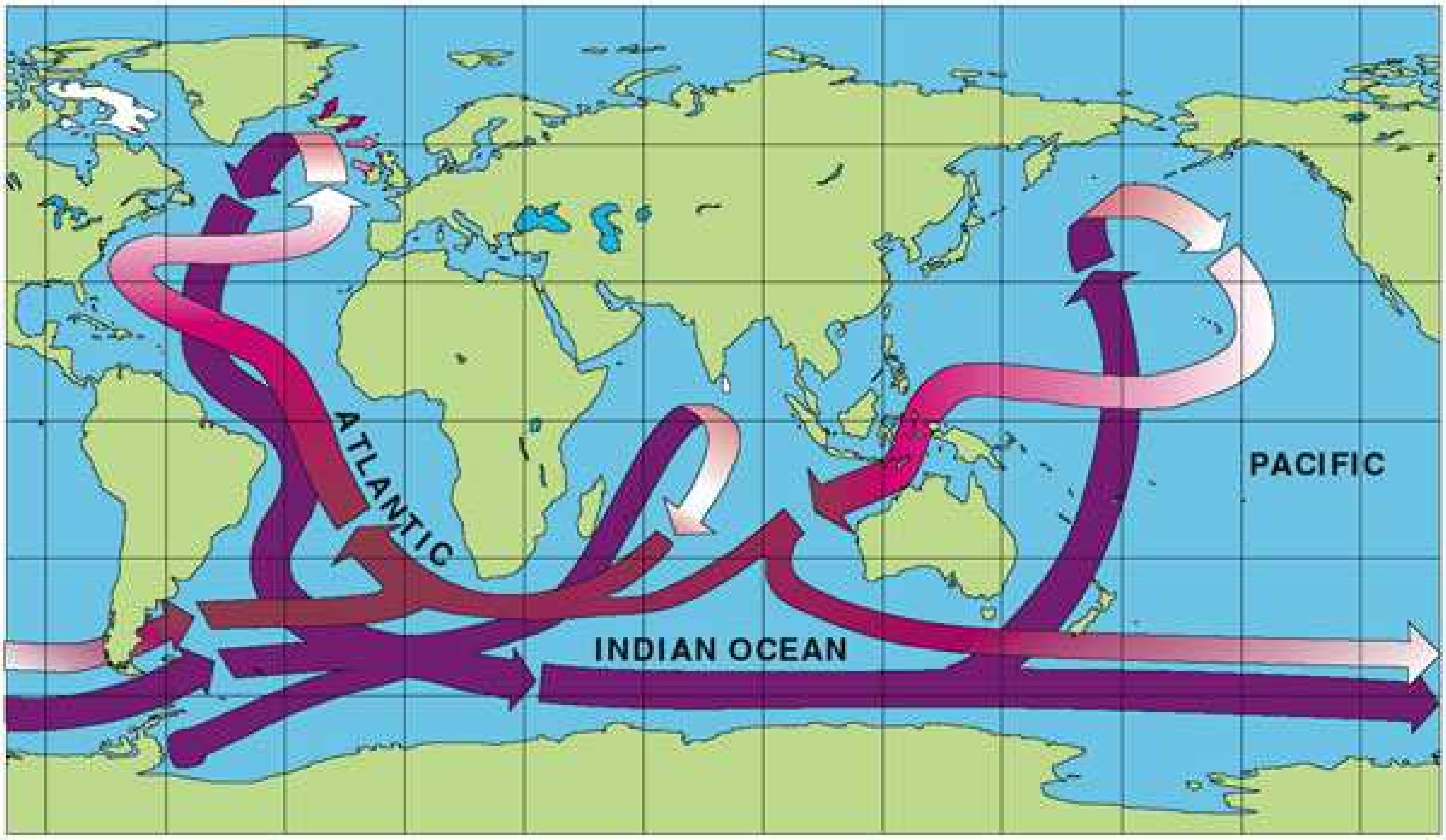
Global circulation



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“ The weight of evidence makes it clear that climate change is a real and present danger. The Exeter conference was told that whatever policies are adopted from this point on, the Earth’s temperature will rise by 0.6F within the next 30 years. Yet those who think climate change just means Indian summers in Manchester should be told that the chances of the Gulf stream - the Atlantic thermohaline circulation that keeps Britain warm - shutting down are now thought to be greater than 50%.”

[**Burying carbon** Leader Column Thursday February 3, 2005 The Guardian]

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- What does this statement mean?
- What analysis was actually done to reach this conclusion?
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- What do we learn about physical systems from the analysis of (necessarily imperfect) models?

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Such analysis results in our Best Current Judgements as to future system behaviour, expressed as uncertainties.

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Stage 3 An analysis so clear and compelling that it would command agreement from all knowledgeable experts.
(This is an objective Bayes analysis (note non-standard use of "objective"!), and the only case where we can talk about, eg THE probability of THC collapse.)

Model analyses in practice

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'Best current judgements' is a high standard to set (though why aim for less?). What we require is care and clarity. This is challenging, but no more challenging, in principle, than the process of collecting system data and building and analysing models themselves. However, this does require a different tool-set and proper resources to carry through.

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To understand the tool-set, we first need to look at the tool-set for individual models, and then consider how this tool-set extends to collections of models.

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Issues Serious assessment of model discrepancy is hard.

The state of the art in climate modelling

Large climate models take months to run on supercomputers. The biggest computer in the world is the Earth Simulator in Japan, which is often used for running climate models.

Leading climate models

One leading climate model at the moment is HadCM3, based at the UK Met Office. One component of this model is HadAM3, the atmospheric module. In a simple experiment to study the effect of CO2-doubling (Murphy et al, 2004, Nature), this is coupled with simple mixed-layer ocean sea-ice models.

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The climate model (HadSM3) has about 100 uncertain parameters, including:

1. *Large scale cloud.* Six parameters
2. *Convection.* Six parameters
3. *Sea ice.* Two parameters
4. *Radiation.* Four parameters
5. *Dynamics.* Four parameters
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We have a few hundred evaluations of HadSM3, made over a period of about three years. These evaluations will be one of the main resources for the UK Climate Impacts Programme 2008 (UKCIP08), which is intended as a fairly definitive statement about how climate change will impact the UK.

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- In particular, input and output very high dimensional and evaluating $f(x)$ for any x may be VERY expensive.

Relating the model and the system

Model evaluations

Actual system

System observations

1. We start with a collection of model evaluations, and some observations on actual system
2. We link the evaluations to the notion of a 'best' evaluation
3. We link the 'best' evaluation to the actual system
4. We incorporate measurement error into the observations
5. There are many useful statistical approaches to assess each individual aspect of uncertainty. The key strength of the Bayesian approach is in the unified treatment of all of the sources of uncertainty.

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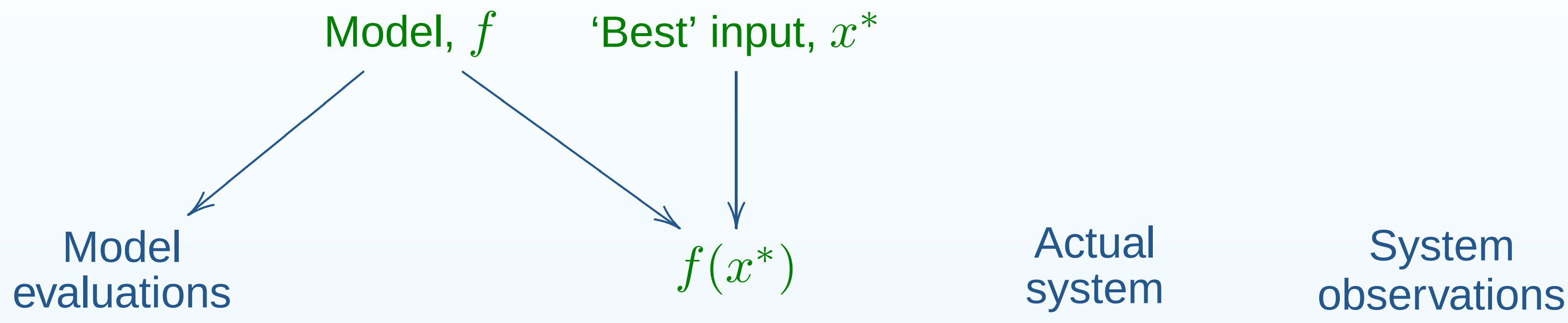

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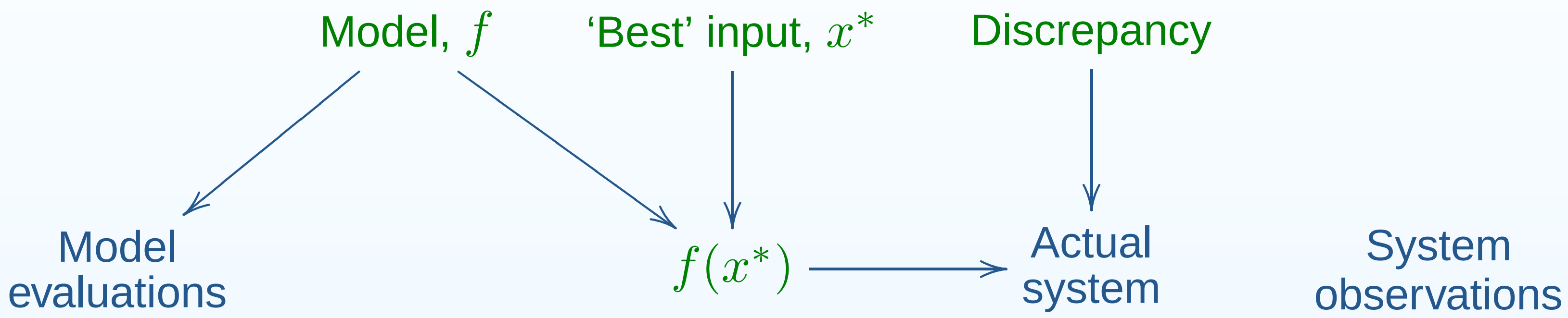

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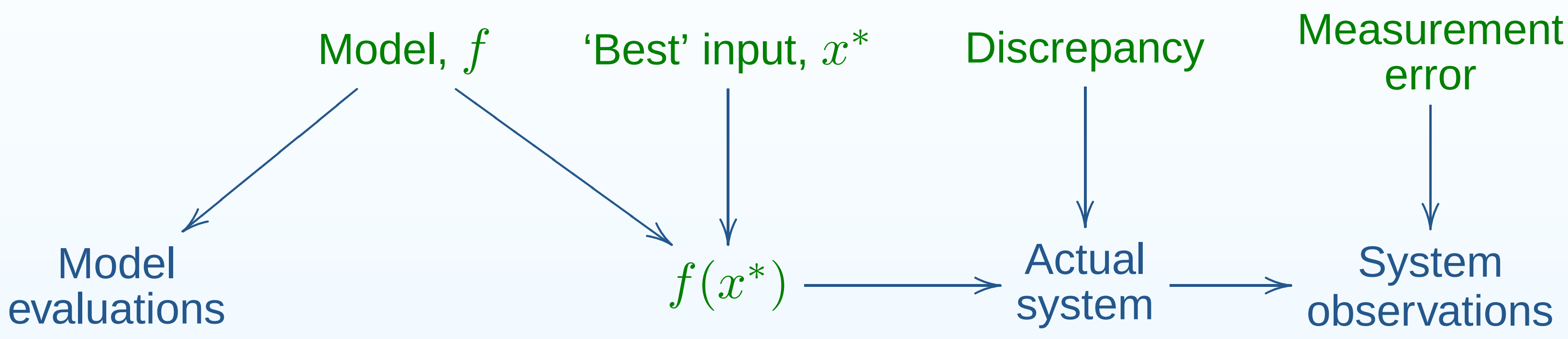

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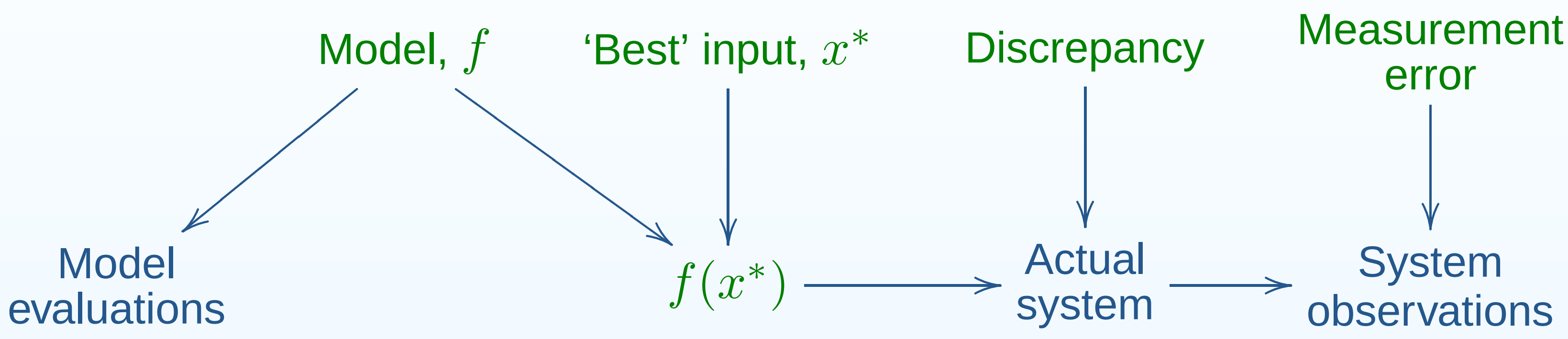

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Representing beliefs about f using emulators

An *emulator* is a probabilistic belief specification for a deterministic function. Our emulator for component i of f might be

$$f_i(x) = \sum_j \beta_{ij}\, g_{ij}(x) + u_i(x)$$

where $B = \{\beta_{ij}\}$ are unknown scalars, g_{ij} are known deterministic functions of x, and $u(x)$ is a weakly stationary stochastic process.
[A simple case is to suppose, for each x, that $u_(x)$ is normal with constant variance and $\mathrm{Corr}(u_i(x), u_i(x'))$ is a function of $\|x - x'\|$.]

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When the input dimension is high, relative to the number of function evaluations we can make, then most of what we may learn about the function comes through the global component. For simplicity, we therefore often suppose that the simulator behaviour can be summarised by the global behaviour (as we dont learn much about local behaviour).

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The emulator expresses prior uncertainty judgements about the function. These are modified by function evaluations. From the emulator, we may extract uncertainty statements for the function, at each input value x, e.g.

$$\mu_i(x) = \mathrm{E}(f_i(x))$$
$$\kappa_i(x, x') = \mathrm{Cov}(f_i(x), f_i(x')),$$

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For problems of medium size, this approach is very powerful and effective.

Bayes linear approach

For very large scale problems a full Bayes analysis is very hard because
(i) it is difficult to make meaningful probability specifications over high dimensional spaces;
(ii) the computations, for learning from data (observations and computer runs), particularly when choosing informative runs, may be technically difficult;
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However, the idea of the Bayesian approach, namely capturing our expert prior judgements in stochastic form and modifying them by appropriate rules given observations, is conceptually appropriate (and there is no obvious alternative).

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(iii) the likelihood surface is extremely complicated, and any full Bayes calculation (based on emulation) may be extremely non-robust.
However, the idea of the Bayesian approach, namely capturing our expert prior judgements in stochastic form and modifying them by appropriate rules given observations, is conceptually appropriate (and there is no obvious alternative).
The Bayes Linear approach is (relatively) simple in terms of belief specification and analysis, as it is based only on the mean, variance and covariance specification which, following de Finetti, we take as primitive.
For a full account, see
Michael Goldstein and David Wooff (2007) Bayes Linear Statistics: Theory and Methods, Wiley.

Calibration via history matching

History Matching is concerned with learning about best inputs, x^*, using simulator evaluations and data, z. Using the emulator we obtain, for each input choice x, the adjusted values of $\mathrm{E}(f(x))$ and $\mathrm{Var}(f(x))$. We rule out regions of x space for which $f(x)$ is likely to be a very poor match to observed z.

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We iteratively refocus on the 'non-implausible' regions of the input space, by further model runs and refitting our emulator over the sub-region and repeating the analysis. This process is a form of iterative global search aimed at finding all choices for x^* which would give acceptable fits to historical data.

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If all values of x are implausible, this is important diagnostic information!

Forecasting without calibration

For the computer model emulator, the mean and variance of $f^* = f(x^*) = Bg(x^*) + u(x^*)$ are obtained from the mean function and variance function of the emulator for $f(x)$. Therefore, we can compute the mean and variance of f^* by conditioning on x^* and then integrating with respect to a prior distribution on x^*.

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Where appropriate, we can improve accuracy by adding a Bayes linear calibration stage to the forecasting (while retaining tractability).

The Best input

How does learning about f inform us about y?

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The simplest (and therefore most popular) way to relate uncertainty about the simulator and the system is the so-called “Best Input Approach”.
We proceed as though there exists a value x^* independent of the function f such that the value of $f^* = f(x^*)$ summarises all of the information that the simulator conveys about the system. This means that we consider the model discrepancy, $\epsilon = y - f^*$, to be independent of f, x^*.

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Further, surprising contradictions arise when we try to construct joint specifications linking collections of models to the physical system in this way.

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[1] Simulator f is informative for y, because f is informative for f^* and $f^*(x^*)$ is informative for y.
[2] A collection of simulators $f_1, f_2, \ldots$ is jointly informative for y, as the simulators are jointly informative for f^*.

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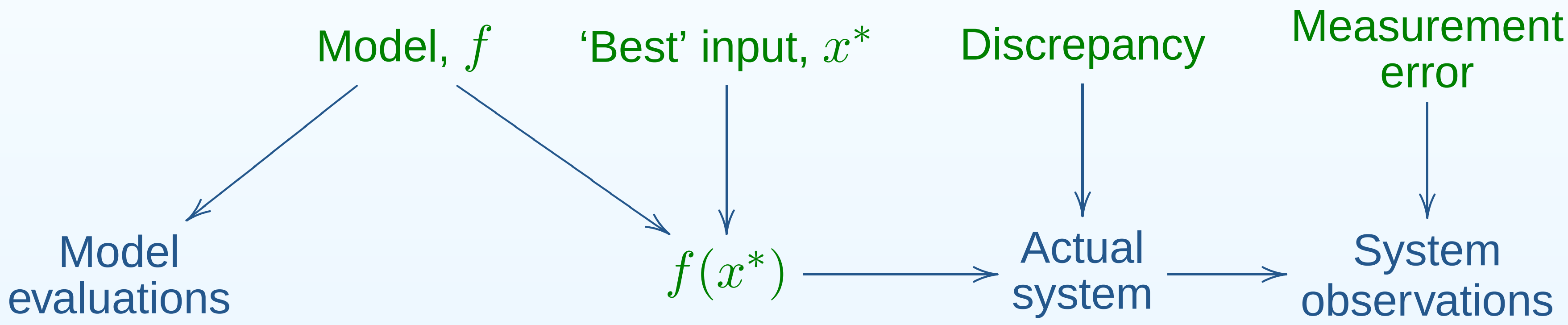

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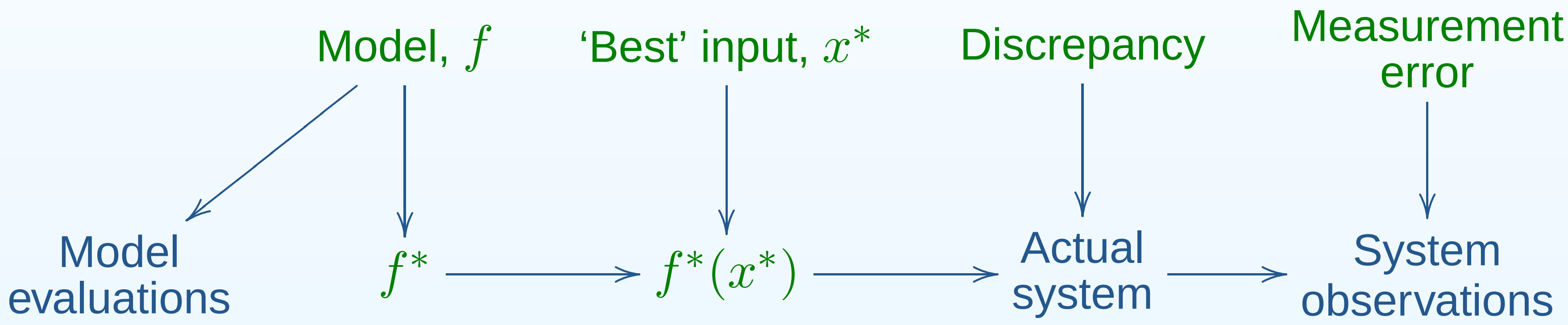

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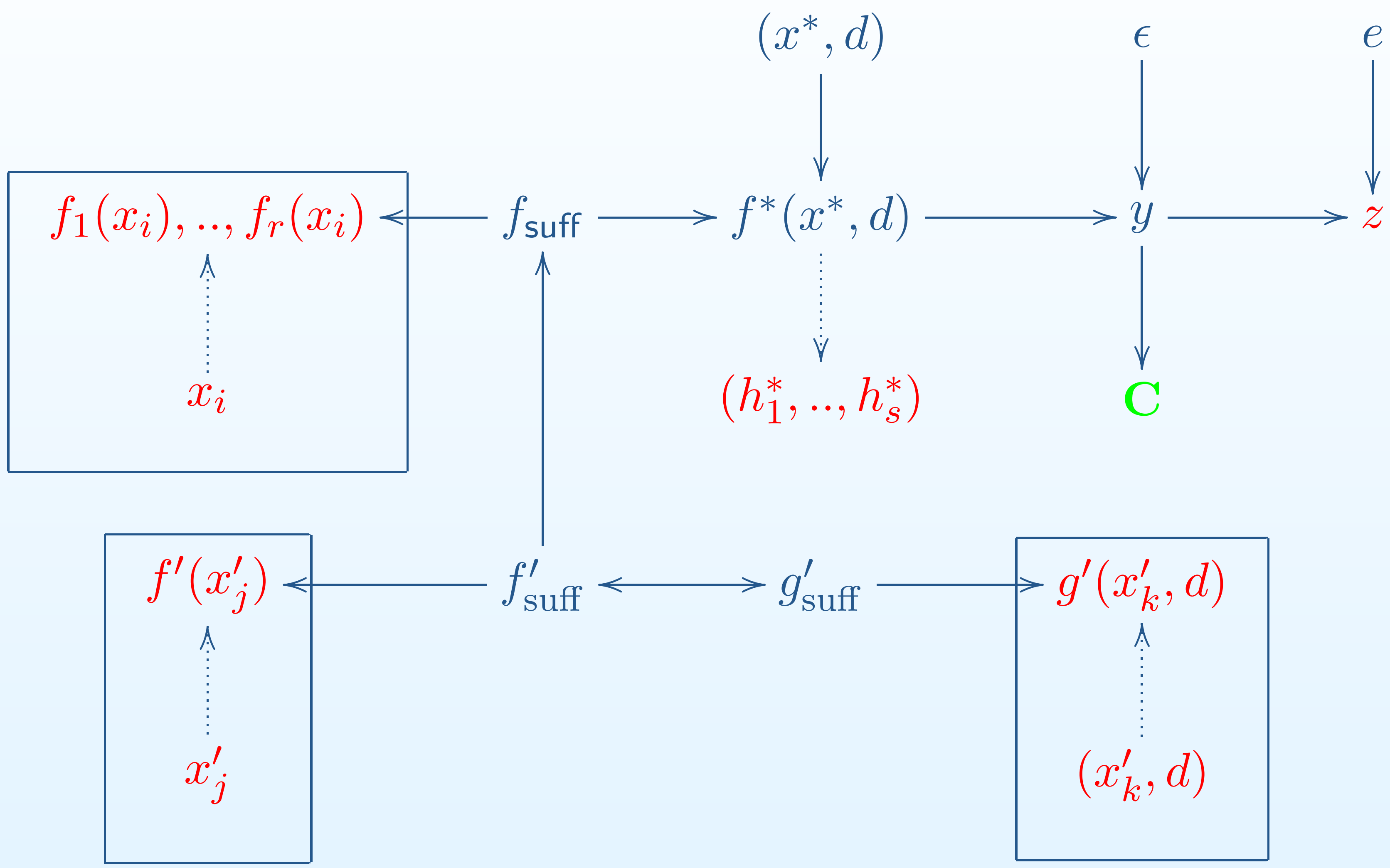
Comment: All our previous methods are unchanged - all that has changed is our description of the joint covariance structure.

A reified influence diagram



Reified simulator f^*; $f_1, \ldots f_k$ are exchangeable refinements of f'; g' is a simulator at level of f' adding decision variables d; (h_i^*) is ensemble of tuned model runs. C is cost to the planet.

Best current judgements for complex systems

To assess best current judgements about complex systems, it is enormously helpful to have an overall framework to unify all the uncertainties arising from

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We have suggested a conceptual/graphical framework for unifying our qualitative and quantitative knowledge about all such uncertainties within a structure which is both logical and tractable, so that we can focus on science rather than technical/computational issues.

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Such analysis poses serious challenges, but they are no harder than all of the other modelling, computational and observational challenges involved with studying large scale physical systems.

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Managing Uncertainty in Complex Models (MUCM) project
[A consortium of Aston, Durham, LSE, Sheffield and Southampton all hard at work on developing technology for computer model uncertainty problems.]