Emulators: huh? what are they good for?

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Managing Uncertainty with Complex Models

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Statistical Emulators

Emulators are used to help explore and understand a computer model.

- They represent our beliefs about the model:
  - if we believe the model displays certain behaviour, then the emulator should too.
- They make probabilistic predictions - should be validated probabilistically, Bastos & O'Hagan 2008.
- Building emulators can't be automated:
  - they need expert input from both the modeller and the statistician.

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- Sensitivity Analysis
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- Calibration/Data assimilation
  - What do the field measurements tell us about the model parameters?
    Kennedy and O'Hagan (2001) JRSS B.
    Higdon, Kennedy, Cavendish, Cafeo & Ryne (2004) SIAM J. Sci. Comp.

Calibrating an EMIC

Use the UVic computer model with a dynamic vegetation module to estimate the sensitivity of respiration and photosysnthesis in plants to changes in CO2 level.

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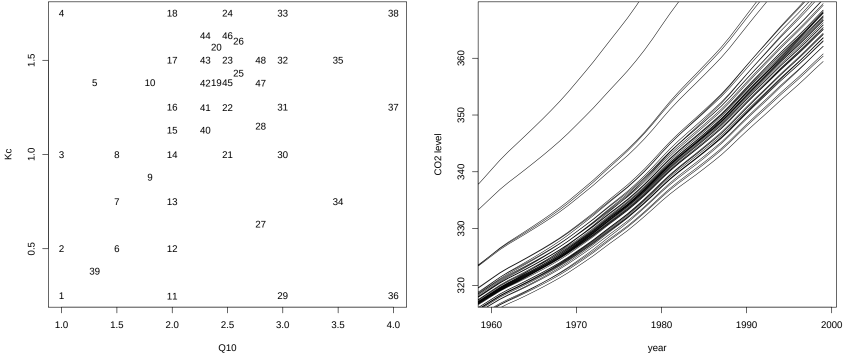

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Kennedy and O'Hagan 2001

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We then aim to find

$$\pi(\hat{\theta}|\mathcal{D}_{sim}, \mathcal{D}_{field}).$$

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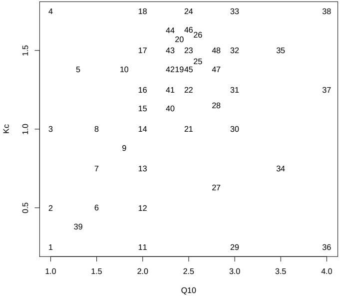

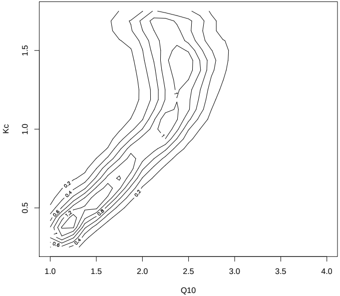

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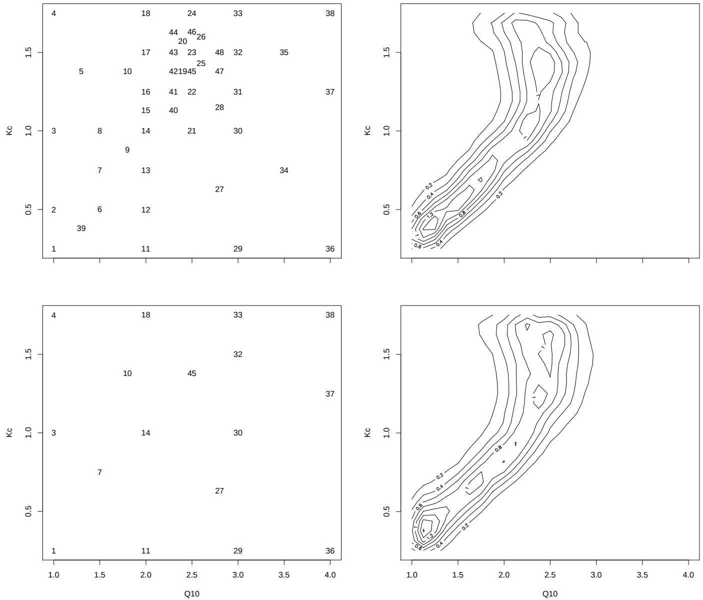

Emulator choices

$$\begin{array}{rcccc} \eta(x) & = & h(x)\beta & + & u(x) \\ \text{emulator} & = & \text{mean structure} & + & \text{residual} \end{array}$$

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- covariance function $c(\cdot,\cdot)$
  - Stationary? Yes - Matèrn, square exponential?
    How to estimate length-scales?
    No - ??

Emulator choices -ctd

How can we build in known model behaviour? For example

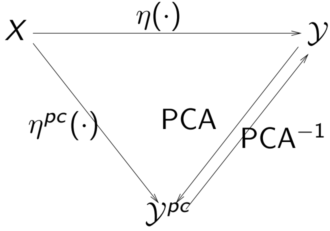
- Positive outputs?
  - transform the data
- Model output obeys physical laws, e.g., conservation of energy?
- Periodicity?
  - careful choice of regressors and covariance function, J.C. Rougier, S. Guillas, A. Maute, A.D. Richmond (2008)

Multivariate output spaces

Climate models typically have many inputs and even more outputs.

How can we build emulators of multivariate models?

- Multivariate emulators.
  - Separable emulator, Conti and O'Hagan 2008 JSPI
  - Outer product emulator, Rougier 2008, J. Comp. Graph. Stats.
- If the outputs are highly correlated, dimension reduction emulators, e.g. EOF emulators
  - Higdon, Gattiker, Williams, Rightle 2008, PNAS.



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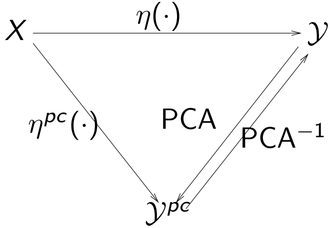

How do we deal with models with a large input space?

- Screening
- Active variable selection?

Conclusions

- Emulators are useful
- Emulators need input from both the modeller and the statistician.